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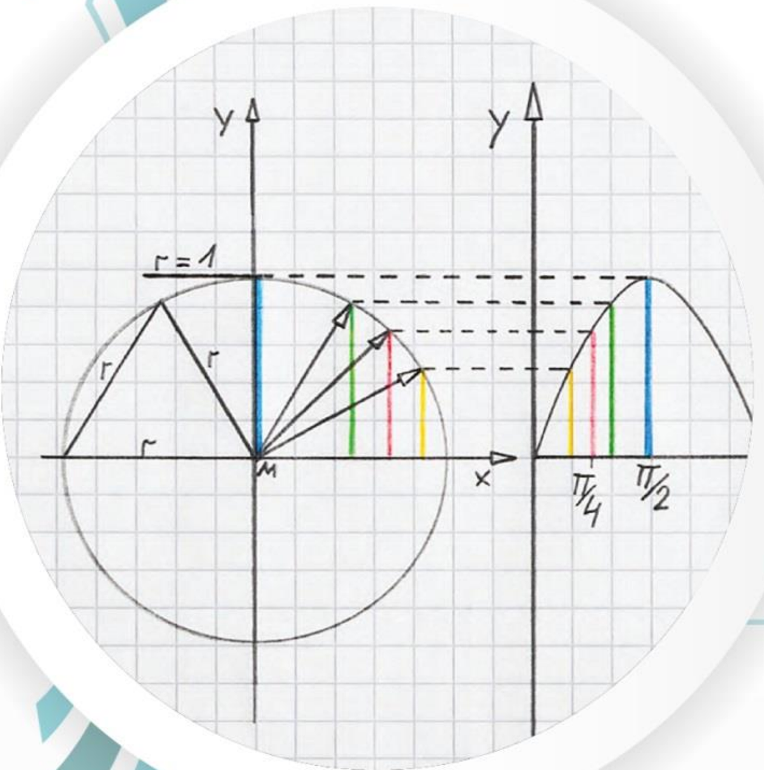

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Application of differential equations in the field of medicine

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Abstract: In this article, doctors describe the processes occurring in the human body, use differential equations to solve medical problems, the law of dissolution of the forms of substances in tablets, determining the amount of the active substance that should be released from the solid dosage form into the solution medium, an example of the breakdown of a drug in the human body, the use of the "Predator-prey" model in the treatment of oncological diseases, the spread of an infectious disease based on the epidemic model.

Keywords: differential equation, predator-prey model, logarithm property, integral, system trajectories, derivative, pharmacopoeia.

The processes taking place in the modern world require in-depth knowledge of specialists and high quality. Today, due to globalization, technology is improving every year, new knowledge and research in the field of medicine is emerging. In order to create technology, people need calculations, which would not be complete without differential equations. In the modern world, activities are changing with the use of mathematical modeling of medical personnel, statistical data and other phenomena used in practice.

We show the use of differential equations in medicine using the simplest mathematical model of an epidemic as an example. The use of differential equations in biology and chemistry also has a medical meaning, because medicine plays an important role in the study of various biological populations (for example, the population of pathogenic bacteria) and chemical reactions in the body (for example, enzymatic reactions).

The role of mathematics in medicine is to aid in the implementation of diagnostic procedures. Currently, the methods of treatment and diagnosis of diseases have been significantly expanded. A significant number of medical centers use mathematical modeling methods, which increase the accuracy of a large number of established diagnoses. Knowing the basics of mathematics is used by doctors to describe the processes occurring in the human body. In many universities, students study mathematics along with basic medical sciences. The main problem of applied mathematics is the selection of an initial mathematical model, which is not felt in any field of knowledge as it is in biology and medicine.

The section "Differential equations" is one of the largest sections of modern mathematics. It intersects with many fields of activity. A differential equation is an equation involving unknown functions, their derivatives of different orders, and arbitrary variables. The same differential equations are widely used in practice, for example, calculating the result of chemical reactions, calculating the main income of the company, current dynamics over time, demographic indicators in a certain region are calculated using differential equations.

The topic of this work will always be relevant, because mathematical methods are used to solve many problems, including in the field of medicine.

Every year, scientists discover more and more new diseases and find treatment methods. And none of this can be done without mathematics.

We will consider the application of differential equations to solve specific medical problems.

1. The law of dissolution of substance forms in tablets.

The "dissolution" test is designed to determine the amount of active substance that should be released from a solid dosage form into a solution medium over a period of time under the conditions specified in the pharmacopoeia article or regulatory documents.

Let n be the remaining amount of the substance in the tablet until the dissolution time t. In it

$$\frac{dn}{dt} = -kn$$

where k is the dissolution rate constant. The minus sign in the equation means that the number of forms of matter decreases over time.

Let's see the solution.

In a differential equation, we isolate the variables and then integrate it:

$$\frac{dn}{n} = -kdt$$

$$\int \frac{dn}{n} = -\int kdt$$

From here:

$$\ln|n| = -kt + \ln|C|$$

Using the logarithm property, we get:

$$|n| = C_1 e^{-kt}$$

here $C_1 = e^c$ arbitrary constant number.

Using the module property, we get:

$$n = C_2 e^{-kt}$$

From here: $C_2 = \pm C_1$ arbitrary constant number

$t = 0$ at $n = n_0$ assuming that at , we $C_2 = n_0$ we get , so:

$$n = n_0 e^{-kt}$$

The formula represents the law of dissolution of dosage forms of the substance from tablets in an integral form. From the equation:

$$n = n_0 e^{-kt}$$

$k -$ we find the melting rate:

$$k = \frac{1}{t}\ln(\frac{n_0}{n})$$

Half-life of tablets $t = t_{\frac{1}{2}}$, $n = \frac{n_0}{2}$

$$\frac{n_0}{2} = n_0 e^{-kt_{\frac{1}{2}}},$$
$$\frac{1}{2} = e^{-kt_{\frac{1}{2}}}$$

We logarithmize both sides of the equation:

$$ln\frac{1}{2} = -kt_{\frac{1}{2}}$$

$t_{\frac{1}{2}}$ we find, $t_{\frac{1}{2}} = \frac{ln2}{k} = \frac{0{,}693}{k}$.

2. An example of drug breakdown in the human body.

Problem: A drug was injected into the patient's body, what part of the drug will be broken down after 8 hours, if 4 mg of the drug has halved its mass after 4 hours?

Solution: To solve this problem, it is necessary to determine the time-dependent change in the amount of medicinal substances in the body. Specify: $N_0 = 8 -$ initial amount of drug (in mg), $N_2 = 4 -$ the amount of the drug after two hours, where N is the amount of the drug at any time. The rate of change of the amount of the drug is proportional to the amount of the drug at a certain time:

$$\frac{dN}{dt} = kn$$

The solution of this differential equation is the following expression describing the desired relationship:

$$N = Ce^{kt}$$

Using the initial conditions, we define C:

$$8 = Ce^{k0}$$
$$e^0 = 1 \text{ from equality}$$
$$C = 8$$

Consequently, $N = Ce^{kt}$. It is known that as soon as the drug was introduced into the body, its mass doubled after 4 hours. k we determine. For this, we put the values t=4,N=4 in the last equation and:

$$4 = 8e^{k4}$$
$$0{,}5 = e^{4k}$$

We logarithmize both sides of the equation:

$$ln0{,}5 = lne^{4k}$$
$$ln0{,}5 = 4klne$$

$lne = 1$ we have the following equality:

$$k = \frac{ln0.5}{4}$$

The dependence of the amount of drug in the body on time can be written as follows:

$$N = 8e^{\frac{ln0.5}{4}t}$$

Now we can find out the amount of substance after 8 hours, for this we insert *t=8* into the equation and get:

$$N = 8e^{\frac{ln0.5}{4}\cdot 8}$$

$$N = 8e^{\ln(0.5)\cdot 2}$$

$ln0{,}5 = -0{,}693$ because $ln(0{,}5)\cdot 2 = -1{,}386$.

As a result:

$$N = 8e^{-1{,}386} = 8\cdot 0{,}25 = 2$$

After 8 hours, the body will have 2 mg of the drug. During this time, 8-2=6 mg was broken down. As a result, it turned out that 6 mg of the substance was broken down in 8 hours.

Currently, the "Prey-prey" model is used in medicine. In the modeling of oncological diseases, tumor cells are considered prey, and lymphocytes, which can suppress tumor motility, are predators. These methods help doctors determine the optimal treatment path and create new ways to fight them.

3. “Predator-prey” model.

$x -$ number of tumor cells,

$y -$ be the number of lymphocyte cells;

If there are changes in the number of tumor cells and lymphocytes over time, then we assume that *x* and *y* are continuous functions of time *t*.

x and y is called the state of the model.

Let's see how the state of the model changes.

$\frac{dx}{dy} -$ the rate of change in the number of tumor cells.

If there are no tumor cells, then the number of lymphocytes decreases. This relationship is linear:

$$\frac{dy}{dt} - a_2 y$$

The rate of change in the number of each species in the ecosystem is also proportional to its number, but only with a coefficient that depends on the number of individuals of another species. Thus, for tumor cells, this coefficient decreases with an increase in the number of lymphocytes, and for lymphocytes, it increases with an increase in the number of tumor cells. This relationship is also linear. Thus, we get a system of differential equations:

$$\begin{cases} \frac{dx}{dt} = a_1 x - b_1 xy \\ \frac{dy}{dt} = -a_2 x - b_2 xy \end{cases}$$

The resulting system of equations is called the Lotka-Volterra model.

$a_1,\ a_2,\ \ b_1,\ b_2 -$ numerical coefficients (model parameters).

The nature of the model state change is determined by the values of (x,y) parameters. By solving this system of equations, it is possible to study the laws of health status change.

When studying a phenomenon, a mathematical model is first created that describes the basic laws that this phenomenon is subject to. In our examples, these laws are expressed in the form of differential equations. Mathematical models facilitate prediction, results of experiments in real systems allow learning.

An epidemic model.

This model describes the spread of an infectious disease in an isolated population. Individuals of the population are divided into three categories. The first class is the x(t)-dimensional infected class (at time t) consisting of infected patients, for whom the incubation period of the infectious disease is assumed to be negligibly short. The second class of numbers is y(t)-susceptible people, that is, people who can be infected by contact with infected patients. Finally, the third class consists of immunocompromised patients, divided into those who develop immunity or die from the disease. Its number is determined by z(t). It is also assumed that the total population n is constant, that is, births, natural deaths and migration are not taken into account. There are two hypotheses based on the model:

1. The incidence at a certain time (t) is x(t)·y(t) - this hypothesis is based on the reasonable assumption that the number of cases is proportional to the number of encounters between sick and susceptible individuals, which in turn is proportional to x. x(t)·y(t); thus, the number of class x increases and the number of class y decreases at the rate a·x(t)·y(t) (a>0);

2) grows proportionally to the number of individuals with immunity, i.e. at the rate b·x(t) (b>0). As a result, we get the following system of equations.

$$\begin{cases} x(t) = a \cdot xy - b \cdot x \\ y(t) = -a \cdot xy \\ z(t) = b \cdot x \end{cases}$$

Assignment. *x'(y)=-1+b/ay* show the.

Due to this problem, it is easy to see that the trajectories of the system have the form shown in Figure 1. Since we are only interested in the positive values of the variables, the equation is unnecessary, since z=n-x-y.

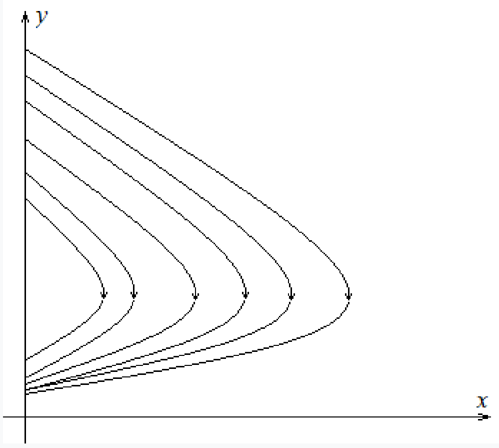

Figure 1. Trajectories of the system

In the presented work, we considered the solution of medical problems using differential equations, for example, the example of the breakdown of a drug in the human body, modeling of the treatment of oncological diseases. The mathematical apparatus for solving differential equations allows solving many problems of the natural science cycle in practice. Higher medical education implies the ability of specialists to apply their knowledge in various medical fields.

We will consider the formation of professional competencies of a specialist by applying mathematical knowledge in medicine.

The application of the knowledge of a medical specialist in the field of health care and medicine includes the implementation of economic activities of a medical organization. The responsibilities of an economist include: preparation of preliminary data for the preparation of economic and financial projects of a medical organization, calculation of the cost of medical services, calculation of necessary materials, development of measures for effective use of capital investments, material, labor and financial resources. An economist is also responsible for increasing labor productivity, reducing the cost of providing medical services, eliminating losses, determining the economic efficiency of labor and production organization, introducing new technologies and inventions, participating in marketing research and predicting the development of a medical organization, etc. should contribute.

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